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(54) Title: INSTRUMENTALITIES FOR INSURING AND HEDGING AGAINST RISK

(57) Abstract

Where frequency and severity of risks, e.g. of catastrophic risks are unknown, as is the case, e.g., for environmental risks, health risks, nuclear reactor risks, and satellite risks, ordinary insurance contracts on occasion are likely to result in claims which an insurer cannot cover. For efficient allocation of risk bearing, an insurance contract is combined or "bundled" with a derivative security. Insurance is contingent on the frequency of the insured event as observed, and the derivative security has a payoff which depends on that frequency. Thus, the derivative security is a contract which is contingent on an index established as a standardized measure of a risk, e.g. of environmental conditions such as El Niño versus La Niña, and/or temperature measures such as heating degree days (HDD), cooling degree days (CDD) and/or precipitation measures in a specific geographic region and during a specified time period.

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INSTRUMENTALITIES FOR INSURING AND HEDGING AGAINST RISK

Technical Field

The invention is concerned with methods, systems and instruments for insuring and hedging against risk, e.g. weather-related risk, including catastrophic risk and other large-scale risk which may be weather-related or otherwise.

Background of the Invention

Concerns are on the rise with large-scale risks, e.g. weather-related risks in view of increasing volatility of weather, climate changes, and population movement to warm coastal areas with attendant changing property prices. Weather affects an estimated \$2 trillion of the \$9 trillion U.S. economy. Such risks are difficult to diversify using traditional insurance and reinsurance practices, and even though climate changes remain putative, the financial challenge is manifest. In the last several years the property/casualty insurance industry has experienced record claims of some \$43 billion in connection with climate volatility. Illustrative of such instances are the Midwest drought of 1988, the Midwest floods of 1993, and flooding along the California coast in 1995. In 1992, Hurricane Andrew caused insured losses of some \$25 billion of which \$18 billion were covered by insurance. In the aftermath of Hurricane Andrew, a significant number of reinsurance companies either stopped offering insurance entirely or limited their offering by ceasing to underwrite catastrophe reinsurance. As reinsurance supply dried up, reinsurance rates increased almost threefold.

In view of insurance inadequacies in covering catastrophic loss, new strategies have been proposed and implemented, e.g. the creation of financial instruments for betting on the frequency of catastrophes which may be weather related or otherwise. Instruments of this type were introduced by the Chicago Board of Trade (CBOT) under the designation of Catastrophe Futures in 1993. Among similar instruments are bonds having a return which is linked to hurricane frequency and severity in the current season and in a specified geographic area. Also, such return has been tied to an insurance company's losses from hurricanes. Typically, the

frequency and severity of catastrophic risks is unknown, as is the case for environmental health risks, health risks, nuclear reactor risks, and satellite risks.

Bilateral contracts contingent on weather risks have been traded by investment banks and brokers/dealers, and offered on a case-by-case basis to insurance companies, energy/utility companies and others whose revenues depend on weather conditions.

Summary of the Invention

To avoid idiosyncratic bilateral contracts, each having a different price for possibly similar services or products, contracts can be based on a standardized index or benchmark. An index can quantify a risk factor to businesses and/or individuals, e.g. atmospheric temperature deviation from a nominal temperature in a specific area and over a specific time interval as expressed by heating degree days (HDD) and cooling degree days (CDD). In correspondence with the quantified risk condition, such an index may be termed "Temperature Index", "Weather Index", "Climate Index" or "El Niño Index", for example.

For hedging against the risk, standardized derivative securities or financial contracts can be drawn contingent on the index, including futures and options, and such contracts can be traded on an exchange, for example.

Furthermore, where insurance is used, for efficient allocation of risk bearing, an insurance contract can be combined or "bundled" with one or more derivative securities. The insurance contract pays an agreed amount contingent on the occurrence of an event, and the derivative securities have a payoff which depends on an index that represents the aggregate frequency of such events, for example.

Brief Description of the Drawing and Appendices

Fig. 1 is a graphic representation of two probability distributions of losses due to hurricanes in El Niño and La Niña years, respectively, in the El Niño Southern Oscillator (ENSO) cycle.

Fig. 2 is a schematic of a technique in accordance with a preferred embodiment of the invention, wherein an insurer executes trades in a financial instrument here designated as ENSO Index or El Niño Index.

Fig. 3 is a flow chart for scientific and computerized determination of heating/cooling degree days.

Fig. 4A is a conceptual diagram for a simple weather contract contingent on a weather index,

Fig. 4B is a conceptual diagram for a call contract on the contract of Fig. 4A.

Included herewith are the appended papers by authors Graciela Chichilnisky and Geoffrey Heal entitled "Managing Unknown Risk" and "Financial Markets for Unknown Risks", respectively.

Detailed Description

The following description is addressed primarily to so-called catastrophe bundles, presupposing a novel risk index and a novel contract contingent on the risk index. For example, if the index is a measure for temperature, the contract will be contingent on a temperature value. While insurance is unsatisfactory when the frequency of a risk is unknown, and securities are unsatisfactory when the risks are individual, a combination of insurance and securities can achieve efficient allocation of risk bearing. Such a combination, here called a catastrophe bundle, can guard against a financial debacle due to overexposure of an insurer, while providing nearly full coverage of the insured. The catastrophe bundle requires the novel risk index which provides a standardized or benchmark measure of the risk, and the novel contract which is contingent on the value of the index. Preferably, the index depends on scientific variables, e.g. temperature or precipitation.

Preferably, a catastrophe bundle is customized based on descriptions of the risk. A computerized mathematical formula can be used in customization of the catastrophe bundle, taking into account a plurality of risk patterns having different actuarial tables. Derivative securities are created with payoffs depending on which description of the risk is applicable, and insurance contracts are created to establish compensation depending on which description of the risk is applicable.

As an example, Fig. 1 shows hurricane incidence depending on the so-called El Niño Southern Oscillator (ENSO) cycle. There are two extreme states of the cycle, known as El Niño and La Niña. In El Niño years, hurricane incidence in the

southeastern part of the United States is below average; in La Niña years it is above average. Fig. 1 shows the probabilities for three outcomes or levels of losses, namely 5, 10 and 15 billion dollars, for El Niño and La Niña years, respectively. As shown, under El Niño conditions the respective probabilities are 0.1, 0.2 and 0.1. Corresponding probabilities are higher under La Niña conditions, namely 0.2, 0.3 and 0.2.

For hedging of an insurance risk in view of the lack of an actuarial function of El Niño versus La Niña conditions, for example, an"ENSO Index" or "El Niño Index" can be used whose value is low under El Niño conditions and high under La Niña conditions. The El Niño Index is an example of an index for a physical parameter, contingent on which a contract can be drawn to pay an agreed amount.

Other environmental indices can be based on precipitation or temperature measures, e.g. heating/cooling degree days for a specific geographic region such as a state or a city, and for a specific period of time. A heating degree day (HDD) is defined for days with an average temperature of less than 65 degrees Fahrenheit, as 65 degrees Fahrenheit minus the daily average temperature. A cooling degree day (CDD) is defined for days with an average temperature of more than 65 degrees Fahrenheit, as the average daily temperature minus 65 degrees Fahrenheit.

Fig. 3 illustrates computerization for determining heating degree days HDD and cooling degree days CDD for an n-day period, e.g., with n=31, for the month of January of a specified year.

Similar indices can be established based on different parameters, e.g. precipitation or yet other climate conditions in a geographic region and for a certain time period.

Contracts contingent on an index can be time-dependent, e.g. with reference to a year, month or any specified time period. For example, contracts can be drawn on cumulative HDDs/CDDs over a time period. Such a contract is an example also of a security which is conditional on the incidence of an insured peril, i.e., on which risk description is applicable.

With the probabilities in accordance with Fig. 1, in an El Niño year the expected value of hurricane damage is calculated in billions as

$$(0.1 \times \$5) + (0.2 \times \$10) + (0.1 \times 15) = \$4$$

Correspondingly calculated, in a La Niña year the expected value is \$7 billion.

The following is under the assumption of a 40% chance of an El Niño year, a 60% chance of a La Niña year and a total value of insured property of \$30 billion. In a worst-case scenario, when hurricane damage is at its maximum of \$15 billion, half of the insured value is at risk.

In an El Niño year the expected loss is 13.33% of the insured risk; in a La Niña year it is 23.33%. Thus, the insurance rates on line, i.e. the premiums as a percentage of the insured amount conditional on being in El Niño and La Niña years would have to be at least 13.33% and 23.33%, respectively, for the insurer to break even in terms of expected value.

But the expected losses are different, depending on the type of year, El Niño or La Niña. Without knowledge of the type of year, the expected loss due to El Niño is the expected loss in an El Niño year times the probability of such a year, i.e. $0.4 \times \$4 = \$1.6$ billion. For La Niña the corresponding calculation is $0.6 \times \$7 = \$4.2$ billion. Hence, without knowledge of the type of year, the expected losses in El Niño and La Niña years are \$1.6 and \$4.2 billion, respectively, for a total of \$5.8 billion.

As to the premiums that would have to be charged for coverage in each type of year without actual knowledge of the type of year, in order to break even on average they would have to be the premiums contingent on being in each year— i.e. 13.33% and 23.33%, respectively— multiplied by the probabilities of each type of year. Thus, without knowledge of the type of year, the rates on line would have to be at least $0.4 \times 13.33\% = 5.33\%$ or $0.6 \times 23.33\% = 13.99\%$, respectively.

If an insurer were to follow conventional procedures of charging premiums based on the over-all expected loss, without distinguishing between the two climate patterns, premiums would be charged to yield the over-all expected loss of \$5.8 billion, implying a rate on line of $5.5/30 = 19.33\%$. This is unsatisfactory, amounting to overcharging in El Niño years when expected claims are \$4 billion and the rate on line need be only 13.33%, and undercharging in La Niña years when expected claims are \$7 billion and rate on line is 23.33%. In the former case, the insurer receives premium income in excess of the expected claims by \$1.8 billion; in the latter, premium income falls short by \$1.2 billion. For proper matching of assets

to liabilities, income from El Niño years should be shifted to La Niña years.

Such shifting can be effected by trading shares of a suitably structured security which is contingent on a novel, standardized index, here termed "ENSO Index" or "El Niño Index" whose value can be related to the incidence of hurricanes, (see Fig.1, for example), and in which traders can take long and short positions. Such trading has the intended effect in that, under the probabilities and the dollar amounts which the insurer has available or will need, the excess \$1.8 billion will be available with a 40% probability and the shortfall \$1.2 billion will be required with a 60% probability, and $0.4 \times \$1.8$ billion = $0.6 \times \$1.2$ billion.

The respective prices of ENSO Index contracts delivering \$1 in both El Niño and La Niña years will be proportional to the probabilities of these events, so that they will be in the ratio of 0.4/0.6 or 2/3. But \$1.2/\$1.8 = 2/3, so that at such prices the sale of surplus income in El Niño years will exactly finance the purchase of income to cover the deficit in La Niña years.

Accordingly, with an ENSO Index, a preferred pattern of financial transactions can be summarized as follows:

1. Issuing insurance contracts which provide coverage against damage in either El Niño or La Niña years.

2. Selling \$1.8 billion of contracts contingent on the ENSO Index having a value corresponding to an El Niño year, at a price of \$0.40 per dollar.

3. Buying \$1.2 billion of contracts contingent on the ENSO Index having a value corresponding to a La Niña year, at \$0.60 per dollar.

By such a combination of trades in securities and insurance policies, here termed a catastrophe bundle, an insurer has complete coverage for themselves as well as their clients, even without knowledge of the odds of loss.

The technique is illustrated by Fig. 2 where t represents a count which starts at t=0 in a year in which insurance is issued for subsequent years t = 1, 2, ...

While the expository example presented above involves just two states (El Niño, La Niña) and three outcomes (losses of \$5, \$10, \$15 billion), a more general derivation can be used to demonstrate the efficacy of catastrophe bundles as follows.

Assuming k states numbered i = 1, ..., k, and n outcomes, numbered j = 1, ..., n, for outcome j the loss will be denoted by x_j, the probability of state i will be

denoted by p_i, and for state i the probability of outcome j will be denoted by p_{ij}.

The probabilities are such that

$$p_i, p_{ij} \geq 0;\ \sum_{\{i\}} p_i = 1;\ \sum_{\{j\}} p_{ij} = 1 \text{ for all } i$$

For outcome j the total probability is

$$q_j = \sum_{\{i\}} p_i\, p_{ij}$$

The mean loss is

$$\mu = \sum_{\{j\}} q_j\, x_j$$

The loss variance is

$$\sigma^2 = \sum_{\{j\}} q_j\, (x_j - \mu)^2$$

For each of the states, the mean loss is

$$\mu_i = \sum_{\{j\}} p_{ij}\, x_j$$

and the corresponding loss variance is

$$\sigma_i^2 = \sum_{\{j\}} p_{ij}\, (x_j - \mu_i)^2 .$$

When an insurer is hedged, with a security, against indeterminacy of the state, the loss variance is

$$\sigma_{CB}^2 = \sum_{\{i\}} p_i\, \sigma_i^2.$$

Without such hedging, the loss variance is

$$\sigma^2 = \sum_{\{j\}} q_j (x_j - \mu)^2$$

$$= \sum_{\{i\}} \sum_{\{j\}} p_i\, p_{ij}\, ((x_j - \mu_i) + (\mu_i - \mu))$$

$$\begin{aligned} = \sum_{\{i\}} p_i \Big[& \sum_{\{j\}} p_{ij} (x_j - \mu_i)^2 + \\ & + 2 \cdot \sum_{\{j\}} p_{ij} (x_j - \mu_i)(\mu_i - \mu) + \\ & + \sum_{\{j\}} p_{ij} (\mu_i - \mu)^2 \Big] \end{aligned}$$

$$\begin{aligned} = & \sum_{\{i\}} p_i\, \sigma_i^2 + \\ & + 2 \cdot \sum_{\{i\}} p_i (\mu_i - \mu) \Big[\sum_{\{j\}} p_{ij}\, x_j - \mu_i \Big] + \\ & + \sum_{\{i\}} p_i (\mu_i - \mu)^2 \cdot \sum p_{ij} \end{aligned}$$

$$= \sigma_{CB}^{\;2} + \sum_{\{i\}} p_i (\mu_i - \mu)^2.$$

Accordingly, the variance with catastrophe bundle (CB) is less than the variance without catastrophe bundle, with the difference being directly related to the magnitudes of the spacing of the μ_i from μ. Therefore, for each expected return, the use of a novel index, novel contract contingent on the index, and novel bundle of insurance and the contract leads to advantageously reduced risk for the expected return.

The following is further with respect to indices, e.g. for weather risk, demand/supply risk, political risk, etc. which are commercially significant in themselves, as are contracts contingent thereon even aside from catastrophe bundles. The use of an index can be sold or licensed by Exchanges such as the New York Stock Exchange, London Stock Exchange and Bermuda Stock Exchange, for example, providing an industry-wide systematic benchmark measure of a specific risk. Contracts which are contingent on such an index can be used for risk hedging or management that protects the revenue of individuals or corporate entities when excessive losses or costs are incurred due to unfavorable conditions, e.g. climate patterns such as El Niño or La Niña, excessively warm or cold periods or excessively

dry or wet periods. Figs. 4A and 4B illustrate such hedging, using a simple contract (Fig. 4A) and a call on the simple contract (Fig. 4B)

Contracts based on indices can be bought/sold jointly with or independently from insurance contracts. They can have one or more "triggers", e.g. HDDs,CDDs, precipitation, time of year or season, length of time, El Niño or La Niña seasons, as well as industry and over-all demand levels for commodities of interest, e.g. electricity, heating oil and natural gas.

Index values can be determined from suitable data, e.g. meteorological, oceanographic, demographic, political or commercial data. Such determinations may involve computational procedures, e.g. accumulating, averaging and smoothing where computerization can be used to advantage to cope with data. Computerization can be used also in trading contracts which are contingent on an index, with buy or sell orders issuing when profitable in view of an actual value of the index as compared with a contract value.

Managing Unknown Risks

The future of global reinsurance.

Graciela Chichilnisky and Geoffrey Heal

It has been said that insurance is the last of the financial services to accept radical change (Denney [1995-1996]). Yet there has been a fundamental shift in the geographic location and in the organization of the reinsurance industry in the last six years (Chichilnisky [1996b]). Global environmental risks are partly responsible for this change; increased weather volatility and catastrophic risks are difficult to diversify using traditional insurance practices.

To provide a map to the future, we need a realistic appraisal of how we got where we are. This is the story of how humans have hedged risks. There are two basic and distinct approaches: statistical and economic. The former is typical of the insurance industry; the latter typifies the securities industry. Both are needed to manage today's catastrophic risks. Neither alone will do. We show how a combination of both leads to efficient outcomes, and is the way to the future (Chichilnisky [1996a, 1996b, 1996d]).

The volatility of weather, taken together with population movement to warm coastal areas and changing property prices, has made catastrophic risks highly unpredictable. Many scientists believe that climate change could be the source. A recent report by the Intergovernmental Panel on Climate Change (IPCC), charged by governments with investigating global warming, says that humans have a "discernible" influence on global climate.

In May 1996, insurance executives confronted the energy industry over global warming, and took their

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case to the United Nations Geneva meeting on climate change in June 1996 (Boulton [1996]). Their case was heard, and for the first time the United States took a leading position in supporting the developing countries' calls for hard targets in the reduction of greenhouse gas emissions in the industrial countries. Environmental markets that trade countries' rights to emit have been proposed and loom large on the horizon.

FINANCIAL RISKS

Although the data on climate change are not conclusive, the financial challenge is already real. In the last few years the property/casualty insurance industry has experienced record claims of about US$43 billion connected with climate volatility. In the United States alone, there was the 1988 Midwest drought, the 1993 Midwest floods, and 1995 flooding along the California coast. Hurricane Andrew in 1992 produced about US$18 billion of insured losses and total losses greater than US$25 billion (Chichilnisky [1996a]).

Andrew was the most devastating natural catastrophe ever recorded. It also led to a wave of financial catastrophe; the hurricane affected almost every insurance company in the United States. Not knowing how to hedge unpredictable risks adds the risk of financial catastrophe on top of that of the natural catastrophe, a one-two punch that could lead to a societal disaster. The year after Andrew, thirty-eight non-U.S. and eight U.S. reinsurers, with names as familiar as Continental Re and New England Re, either withdrew from the business or ceased underwriting catastrophe reinsurance (Chichilnisky [1996b]).

Facing an impossible challenge, many reinsurers left the market. Worldwide reinsurance capacity dropped more than 30% between 1989 and 1993, and it appears that over 20% of that is due to Andrew. This naturally led to changes in the marketplace. Insurance companies could not buy enough catastrophe reinsurance, no matter how hard they tried. As supply dried up, prices of course increased dramatically; the rate on line went from 6.2% in 1989 to 21.4% in 1994.

Higher prices then attracted new capital. This led to a major geographic shift of the industry. Continuing doubts about the future existence of Lloyd's of London led to a drop in the U.K. market share, from about 56% in 1989 to 23% in 1995. Since 1993 Bermuda's reinsurance industry evolved from practically zero to its current position of 25% of the market. Investment banks are now betting heavily on the reinsurance market. They are the owners of most of the businesses created since 1992.

REVOLUTION IN GLOBAL FINANCE

Together with the geographic shift, there has been a substantial shift in the industry's strategy. The insurance derivatives that have been recommended for several years are starting to play a role.

In 1992, we recommended the creation of an instrument to bet on the frequencies of catastrophes, which the Chicago Board of Trade (CBOT) introduced under the name Catastrophe Futures in 1993 (see Chichilnisky and Heal [1993]). In 1997, Morgan Stanley started marketing a similar instrument: a bond issue whose returns are linked to hurricane frequency and severity in the current U.S. season. Recently, Merrill Lynch structured a transaction for USAA, the country's largest direct marketer of home and car insurance, offering US$500 million in bonds on the U.S. capital markets that are tied to the company's losses from hurricanes (see Waters [1996]).

Financial innovation in reinsurance markets is slowly developing, but the underlying pressure is relentless. Everyone knows that access to more liquid capital markets is essential to the reinsurance industry. The derivatives market is the key to liquid and flexible trading of weather risks.

UNKNOWN RISKS

Unknown risks are risks whose frequencies we do not know, and for which we are aware of our ignorance (Chichilnisky [1996d]). You could think of these as risks for which we have more than one actuarial table, each equally likely. There is more than one prior estimate of the frequency of the event (see Cass, Chichilnisky, and Wu [1996]).

Examples of unknown risks are environmental health risks of new and little known epidemics, or risks induced by scientific uncertainty in predicting the frequency and severity of catastrophic events such as nuclear reactor and satellite risks. These risks are driving major changes in the insurance and reinsurance industry today (see Chichilnisky and Heal [1998]).

Take a simple example. One reliable source gives a 2% annual chance of the occurrence of a hurricane of a certain type, and another a 12% chance. Monte Carlo

simulations and other procedures can be used to attempt to tease from all models a unique statistical approximation to the true frequency. But what if there is no true frequency?

How could this be? Easily. There may be two possible climate patterns, both equally likely. This is typical of complex and chaotic systems such as the climate (see Chichilnisky [1998]).

Many climate experts view climate as a fundamentally non-linear phenomenon in which chaotic patterns emerge easily. Such systems can have two "attractors," or two distinct overall patterns of behavior, each significantly likely. Each of these attractors describes a weather pattern, a reasonable statistical inference of the frequencies of a major event. In such a chaotic system, it is scientifically impossible to predict from the initial conditions which of the two patterns the climate will take: a pattern with two hurricanes a year, or the other with a dozen. Because we cannot predict, we face a risk. We call it a chaotic risk because it emerges from the chaotic nature of the climate system.

The first statistical reaction is to construct a new actuarial table by taking an average; assuming the two states, 2% and 12%, are equally likely, this is 7%. But taking an average does not help. It only ensures that one is wrong 100% of the time: 50% of the time we are overinsured (the pattern with two hurricanes per year), and the other 50% we are underinsured (the pattern with a dozen a year). Both have major financial costs. If each hurricane leads to US$2 billion in losses, the averaging method leads to a US$10 billion shortfall 50% of the time and US$10 billion overinsurance the other 50% of the time. Hardly a measured way to manage risks.

Is there a solution to this problem? The good news is that there is. It is possible to hedge such unknown risks successfully and efficiently. To do so, however, one needs a careful and customized approach that blends both insurance and securities approaches to hedging risks.

TWO WAYS TO HEDGE RISK

Insurance: The Statistical Approach

The *statistical* approach to hedging risks, which relies on the law of large numbers, is the traditional foundation of the insurance industry.

For this to work, risks must be reasonably independent across individuals or groups, and the frequencies must be known. Loss of life and car accidents are typical examples. Here the law of large numbers operates.

There is safety in numbers; with a large enough population, the number of those likely to be affected is known with considerable accuracy. The sample mean is highly predictable if the distribution for each person or group is known. This is the standard principle on which insurance operates.

Reinsurance is simply a way to augment the pool of those affected so that the law of large numbers operates better. All that is needed is a reliable actuarial table describing the incidence per person or group, and a large pool of insureds to distribute the risk (see Chichilnisky and Heal [1993]).

If the numbers are not large enough, it is standard to spread risk through time. The number of people affected by a hurricane over a ten-year period is at least ten times that affected in one year. This requires that the risks be independent through time, eliminating irreversible risks such as once-and-for-all shifts arising from global warming.

Hurricanes such as Andrew (1992) and Opal (1995), however, defy the law of large numbers. They affect large areas all at once, both in physical and in financial terms, and their frequency and severity seem to be changing. The actuarial table itself has become the risk. Insurance does not work. What are the alternatives?

Derivatives: The Economic Approach

An alternative is the *economic* approach. This works best for correlated risks, in which the same event occurs for many people all at once. A drop in the value of the dollar is an example; the event is the same for everyone in the U.S. economy. There is no way to *pool* this risk, although, as we all know, we can hedge it by using derivatives (currency futures or options). The principle used here is negative correlation. One hedges by taking a position that is highly correlated with the risk, except with the opposite sign.

For example, an investor with a dollar-based portfolio who fears a drop in the value of the dollar can buy a futures contract in yen, or a dollar put. If the dollar drops in value, the investor is covered by the increase in the value of the derivative. Bear funds have been constructed on this principle.

The economic procedure is radically different from the insurance approach in that it does not require a large number of people. Nor does it require knowing the frequency of the event or the actuarial table. This funda-

mentally different method is the way the securities industry operates. Instead of *pooling* risks, one *trades* risks.

Securities markets are, however, notoriously complex. For example, the procedure of trading risks just outlined makes no sense for individual risks, such as death. How would we describe the death of one single person within a large economy as one event on which all of us can trade? To do so would require an unrealistically high number of securities, indeed 2^x, where x is the number of people in the economy. In a world with five billion people, the number of securities could exceed the number of all known particles in the universe (see Chichilnisky and Heal [1998]).

Insurance, instead, deals with such risks expeditiously. If all individuals are in a similar risk class, one insurance contract would suffice. The contrast is stark, but it makes a point. In a world of unknown risks, neither securities nor insurance methods work in isolation.

THE IDEAL HEDGE: CATASTROPHE BUNDLES

We see that insurance does not work when the frequency of a risk is unknown, and securities do not work when the risks are individual. If neither of these two approaches works on its own, what does work?

The ideal hedge is a combination of insurance and securities; this can achieve efficient allocation of risk-bearing. We call this a *catastrophe bundle* because it bundles together two types of instruments. It consists of an insurance instrument with a novel derivative security for betting on the frequency itself (see Chichilnisky and Heal [1993]).

The latter type of security has emerged and is now traded on the CBOT. As we have mentioned, related securities have recently emerged also in the form of bonds floated by Morgan Stanley and Merrill Lynch.

The combination of both instruments ensures that no financial catastrophe will occur, since the reinsurer is not exposed to more risks than it can afford. At the same time, this approach can be used to provide nearly full coverage for the insured at a minimal cost.

We show elsewhere that such instruments lead to an efficient allocation of risk-bearing (see Chichilnisky and Heal [1993, 1998] and Cass, Chichilnisky, and Wu [1996]). They require a carefully customized approach to hedging risk. This gives the traditional face-to-face insurance approach an edge over raw technology.

HOW DO CATASTROPHE BUNDLES WORK?

Catastrophe bundles work best in the hands of an experienced reinsurer or broker who can customize the instrument to the client's needs. In a way, the reinsurer is selling a package that consists of insurance, a security, and a risk management/consulting tool.

The broker must first identify with the client the set of possible descriptions of the risk. This crucial part of the process involves new techniques of risk management. It is best handled on a face-to-face and customized basis. A mathematical formula is then brought to bear in customizing catastrophe bundles to customer needs. This formula works very well when there is more than one pattern of risk and therefore more than one "possible" actuarial table, each table being substantially likely.

After this is achieved, derivative securities whose payoffs depend on which description of the risk is correct are introduced. These securities serve to hedge uncertainty about actuarial tables. Finally, one structures insurance contracts that establish a compensation arrangement in a way that depends on which description of the risk is correct.

Catastrophe bundles are proprietary, and their use in a particularly simple case is illustrated in Exhibit 1.

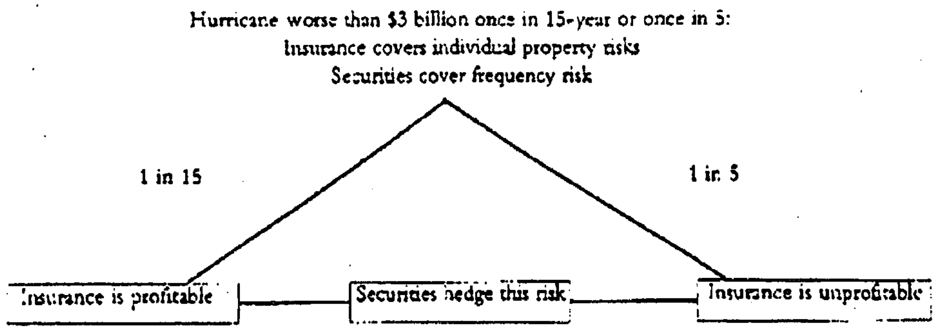
PRICING AND OPTIMAL PORTFOLIOS

Fund managers can look at the flip side of this picture and seek a combination of insurance and securities that offer an optimal portfolio in insurance and investment markets. A part of this instrument is what Merrill Lynch and Morgan Stanley have floated recently. Securitizing such instruments is, of course, the next step.

Through the use of catastrophe bundles, the reinsurance broker can access a large pool of managed funds while offering its clients a customized reinsurance service that manages risks optimally, and at very competitive prices.

Pricing, of course, is a crucial issue. What is needed here is to separate two parts of the risks and to push each as far as it will go. The contingent insurance part of the instrument should be applied as far as possible, covering the independent part of the risk for which it is optimally suited. Securities are then used for the purpose for which they are best: the correlated part of the risk. A mathematical formula used to construct the catastrophe bundle separates and prices both parts.

EXHIBIT 1
CATASTROPHE BUNDLE EXAMPLE



CONVERGENCE OF INSURANCE AND SECURITIES MARKETS

It is no secret that the securities industry is making inroads into the reinsurance business. By itself, however, it cannot succeed, because the individual parts of the risks cannot be handled efficiently by securities markets: they are too cumbersome for individual risks. Insurance, based on the law of large numbers, has an important place in simplifying financial transactions and hedging known individual risks.

Catastrophe bundles offer one approach to computing the limits of each instrument and blending them optimally to achieve the most competitive pricing of a catastrophe reinsurance portfolio.

The future of the industry is in the hands of those who achieve the optimum balance, through integrating derivative securities with contingent insurance contracts, and integrating technology with customized face-to-face know-how.

HURRICANE RISKS AND EL NIÑO: AN EXAMPLE

How exactly would catastrophe bundles work? We answer that question with a simple but typical example, drawn from hurricane insurance. Hurricane incidence is conditioned by the ENSO cycle, so we consider, instead of hurricane bonds of the type that have recently been issued, a tradable ENSO index.[2] This index would achieve everything one needs from hurricane bonds, but in a more general and simple fashion.

A tradable ENSO index is a contract that pays an agreed amount contingent on the value of a physical index. It is similar in concept to the catastrophe futures traded on the CBOT, and is an example of a security conditional on the incidence of the insured peril, that is, on which risk description is correct.

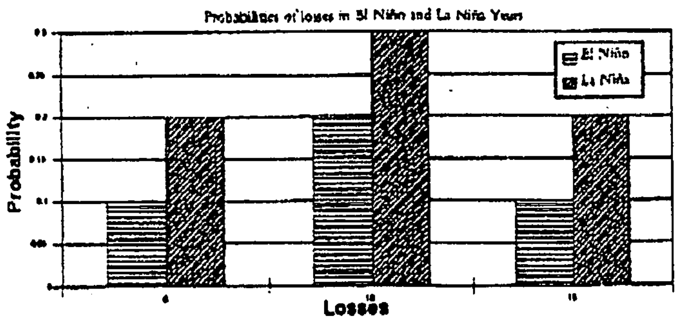
There are two extreme states of the ENSO cycle, known as El Niño and La Niña. In El Niño years, hurricane incidence in the southeastern U.S. is below average; in La Niña years, it is above.

Exhibit 1 shows possible probability distributions of damage due to hurricanes conditional on El Niño or La Niña years.

As an example, assume that, in an El Niño year, there is a 10% chance of a $5 billion loss, a 20% chance of a $10 billion loss, and a 10% chance of a $15 billion loss. The expected value of the damage is therefore (0.1 × $5) + (0.2 × $10)+(0.1 × $15) = $4 billion. In a La Niña year, the probabilities are 20%, 30%, and 20%, respectively, giving an expected loss of $7 billion. Assume that there is a 40% chance of an El Niño year, and a 60% chance of a La Niña year.[3] The total value of insured property is taken as $30 billion, so that in a worst case scenario — when the hurricane damage is at its maximum of $15 billion — half of this value is at risk.

In an El Niño year, the expected loss is 13.33% of the insured risks, and in a La Niña year, it is 23.33%. It follows that the rates on line (i.e., premiums as a percentage of the insured amount) conditional on being in El Niño and La Niña years would need to be at least 13.33% and 23.33%, respectively, to break even in expected value terms.

EXHIBIT 2
HURRICANE PROBABILITIES AND THE ENSO SYSTEM



As we have already noted, expected losses are different, depending on what type of year we are in. Before we know what kind of year will occur, we therefore have an expected loss due to El Niño equal to the expected loss in an El Niño year times the probability of such a year, i.e., (0.4 × \$4) = \$1.6 billion. For La Niña, the equivalent calculation is (0.6 × \$7) = \$4.2 billion. Hence, ex ante, before we know which year we are or will be in, the expected losses in El Niño and La Niña years are, respectively, \$1.6 billion and \$4.2 billion, giving a total of \$5.8 billion as the annual expected loss altogether.

We can now compute the premiums that would have to be charged for cover in each type of year before the type of year is known, in order to break even on average. These would have to be the premiums contingent on being in each year — seen above to be 13.33% and 23.33% for El Niño and La Niña — multiplied by the probabilities of each type of year. Thus the ex ante rates on line (before it is known whether we are in an El Niño or a La Niña year) have to be at least (0.4 × 13.33%) = 5.33% or (0.6 × 23.33%) = 13.99%, respectively.

If insurers follow the obvious and traditional procedure of charging premiums based on the overall expected loss and not distinguishing between the two climate patterns, they will charge premiums that will bring in their overall ex ante expected loss of \$5.8 billion, implying a rate on line of 5.8/30 = 19.33%. This is unsatisfactory because in El Niño years they are overcharging (expected claims are \$4 billion; the rate on line need be only 13.33%); La Niña years, they are undercharging (expected claims are \$7 billion; a rate on line of 23.33% is needed).

In the former case, the insurers are charging premiums in excess of expected losses by \$1.8 billion, hardly a competitive strategy, and in the latter case, premium income falls short of expected claims by \$1.2 billion, clearly a dangerous and unsustainable position. Neither case is satisfactory. To match assets to liabilities properly, insurers need to shift income from El Niño to La Niña years.

This is where securities conditional on incidence, on description of the risk, come into the picture. They can be used to transfer income between El Niño and La Niña years so that the surplus in the former cover the deficit in the latter. We need a security whose value depends on the incidence of hurricanes; for the purposes of this example, we take this to be a tradable ENSO index. This would be a contract whose value depends on the value of the ENSO index and in which traders can take long or short positions. By trading this security, the insurer in our example can in effect trade income in El Niño years for income in La Niña years.

The odds work out nicely. The insurer wants to sell \$1.8 billion in an El Niño year, its surplus of premium income over expected claims, which occurs with a 40% chance. Correspondingly, it needs to buy \$1.2 billion of income in La Niña years, to cover the shortfall between premium income and expected claims. In our example, this happens 60% of the time.

The prices for ENSO index contracts delivering \$1 in El Niño and La Niña years will be proportional to the probabilities of these events, and so will be in the ratio of 0.4/0.6 or 2/3. But \$1.2 billion/\$1.8 billion = 2/3, so that at such prices the sale of surplus income in El Niño years will exactly finance the purchase of income to cover the deficit in La Niña years.

Overall, then, we have a pattern of transactions as follows:

1. Issuing insurance contracts which provide cover against damage in either El Niño or La Niña years.
2. Selling \$1.8 billion of contracts contingent on the ENSO index having a value corresponding to an El Niño year, at a price of \$0.40 per dollar.
3. Buying \$1.2 billion of contracts contingent on the

ENSO index having a value corresponding to a La Niña year, at $0.60 per dollar.

This specific combination of trades in securities and insurance policies described in these steps is what we refer to as "catastrophe bundles." Through trading catastrophe bundles, insurers can arrange complete cover for themselves and their clients at minimum cost, in spite of not knowing what the odds of loss will be. They achieve this by a specific tailor-made combination of insurance contracts and securities. All these contracts are conditional on the incidence of the insured risk.

How different is this approach from the practice today? The securities issued today securitize insurance or reinsurance risks, and therefore bring more liquidity to the reinsurance market. This is an improvement. But these securities still leave open the possibility that the insurer is either offering non-competitive rates or taking on a dangerous exposure. Today's securities do not tackle the essence of the problem.

The key to catastrophe bundles is to recognize that when there are several possible actuarial tables, all reasonably likely, we have to supplement insurance introducing and trading securities dependent on them. A specific combination of insurance and securities, and an equally specific pricing policy, are required for an optimal allocation of risks on competitive terms.

ENDNOTES

[1]Chichilnisky (1996c) advances a proposal for a global market on greenhouse gas emissions and an International Bank for Environmental Settlements to handle executions, clearing, and settlements as well as regulate borrowing and lending rates.

[2]ENSO stands for the El Niño-Southern Oscillator, the name given to the weather pattern that originates in the equatorial Pacific and influences rainfall and storm incidence from Australia to southern Africa. An indicator of the state of the ENSO cycle is a sea surface temperature (SST) index for the equatorial Pacific.

[3]This is a simplification. There are also years that are neither, so-called neutral years. The numbers we use in this example are purely illustrative.

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GRACIELA CHICHILNISKY AND GEOFFREY HEAL

3.5. Financial Markets for Unknown Risks *

1. Introduction

New risks seem to be an unavoidable in a period of rapid change. The last few decades have brought us the risks of global warming, nuclear meltdown, ozone depletion, failure of satellite launcher rockets, collision of supertankers, AIDS and Ebola.[1] A key feature of a new risk, as opposed to an old and familiar one, is that one knows little about it. In particular, one knows little about the chances or the costs of its occurrence. This makes it hard to manage these risks: existing paradigms for the rational management of risks require that we associate probabilities to various levels of losses. This poses particular challenges for the insurance industry, which is at the leading edge of risk management. Misestimation of new risks has lead to several bankruptcies in the insurance and reinsurance businesses.[2] In this paper we propose a novel framework for providing insurance cover against risks whose parameters are unknown. In fact many of the risks at issue may be not just unknown but also unknowable: it is difficult to imagine repetition of the events leading to global warming or ozone depletion, and, therefore, difficult to devise a relative frequency associated with repeated experiments.

A systematic and rational way of hedging unknown risks is proposed here, one which involves the use of securities markets as well as the more traditional insurance techniques. This model is quite consistent with the current evolution of the insurance and reinsurance industries, which are beginning to explore the securitization of some aspects of insurance contracts via Act of God-bonds, contingent drawing facilities, catastrophe futures and similar innovations. In fact, our model provides a formal framework within which such moves can be evaluated. An earlier version of this framework was presented in [6]; Chichilnisky [3] gives a more industry-oriented analysis.

This merging of insurance and securities market is not surprising: traditionally economists have recognized two ways of managing risks. One is risk

* We are grateful to Peter Bernstein, David Cass and Frank Hahn for valuable comments on an earlier version of this paper.

G. Chichilnisky et al. (eds.), Sustainability: Dynamics and Uncertainty, 277–294.

pooling, or insurance, invoking the law of large numbers for independent and identically distributed (IID) events to ensure that the insurer's loss rate is proportional to the population loss rate. This will not work if the population loss rate is unknown. The second approach is the use of securities markets, and of negatively correlated events. This does not require knowledge of the population loss rate, and so can be applied to risks which are unknown or not independent. In fact, securities markets alone could provide a mechanism for hedging unknown risks by the appropriate definition of states, but as we shall see below this approach requires an unreasonable proliferation of markets. Using a mix of the two approaches can economize greatly on the number of markets needed and on the complexity of the institutional framework. In the process of showing this, we also show that under certain conditions the market equilibrium is anonymous in the sense that it depends only on the distribution of individuals across possible states, and not on who is in which state.

The reason for using two types of instrument is simple. Agents face two types of uncertainty: uncertainty about the overall incidence of a peril, i.e., how many people overall will be affected by a disease, and then given an overall distribution of the peril, they face uncertainty about whether they will be one of those who are affected. Securities contingent on the distribution of the peril hedge the former type of uncertainty: contingent insurance contracts hedge the latter.

Our analysis implies that insurance companies should issue insurance contracts which depend on the frequency of the peril, which we call a statistical state. The insurance companies should offer individuals an array of insurance contracts, one valid in each possible statistical state. Insurance contracts are, therefore, contingent on statistical states. Within each statistical state, of course, probabilities are known. Therefore, companies are writing insurance only on known risks, something which is actuarially manageable. Individuals then buy the insurance that they want between different statistical states via the markets for securities that are contingent on statistical states. The following is an illustration for purchasing insurance against AIDS, if the actuarial risks of the disease are unknown. One would buy insurance against AIDS by (1) purchasing a set of AIDS insurance contracts each of which pays off only for a specified incidence of AIDS in the population as a whole, and (2) making bets via statistical securities on the incidence of AIDS in the population. Likewise, one would obtain cover against an effect of climate change by (1) buying insurance policies specific to the risks faced at particular levels of climate change, and (2) making bets on the level of climate change, again using statistical securities. The opportunity to place such bets is currently provided in a limited way by catastrophe futures markets which pay an amount depending on the incidence of hurricane damage.

The present paper draws on recent findings of Chichilnisky and Wu [5] and Cass et al. [4], both of which study resource allocation with individual risks.

Both of these papers develop further Malinvaud's [15, 16] original formulation of general equilibrium with individual risks, and Arrow's [1] formulation of the role of securities in the optimal allocation of risk-bearing. Our results are valid for large but finite economies with agents who face unknown risks and who have diverse opinions about these risks: in contrast, Malinvaud's results are asymptotic, valid for a limiting economy with an infinite population, and deal only with a known distribution of risks. Our results use the formulation of incomplete asset markets for individual risks used to study default in [5, section 5.c]. The risks considered here are unknown and possibly unknowable, and each individual has potentially a different opinion about these risks, while Chichilnisky and Wu [5] and Cass et al. [4] assume that all risk is known.

2. Notation and Definitions

Denote the set of possible states for an individual by S, indexed by $s = 1, 2, \ldots, S$. Let there be H individuals, indexed by $h = 1, 2, \ldots, H$. All households have the same state-dependent endowments: endowments depend solely on the household's individual state s, and this dependence is the same for all households. The probability of any agent being in any state is unknown, and the distribution of states over the population as a whole is also unknown. A complete description of the state of the economy, called a *social state*, is a list of the states of each agent. There are S^H possible social states. A social state is denoted σ : it is an H-vector. The set of possible social states is denoted Ω and has S^H elements. A statistical description of the economy, called a *statistical state*, is a statement of the fraction of the population in each state: it is an S-vector. There are $\binom{H+S-1}{S-1}$ statistical states. Clearly many social states map into a given statistical state. For example, if in one social state you are well and I am sick and in another, I am well and you are sick, then these two social states give rise to the same statistical state. Intuitively, we would not expect the equilibrium prices of the economy to differ in these two social states. One of our results shows that under certain conditions, the characteristics of the equilibrium are in fact dependent only on the statistical state.

How does the distinction between social and statistical states contribute to risk management? Using the traditional approach, we could in principle trade securities contingent on each of the S^H social states. Clearly this would require a large number of markets, a number which grows rapidly with the number of agents. The institutional requirements can be greatly simplified. When the characteristics of the equilibrium depend only on the statistical state, one can trade securities which are contingent on statistical states, i.e., contingent on the distribution of individual states within the population, and still attain efficient allocations. We will trade securities contingent on whether

4 or 8% of the population are in state 5, but not on which people are in this state. Such securities, which we will call *statistical securities*, plus mutual insurance contracts also contingent on the statistical state, lead (under the appropriate conditions) to an efficient allocation of risks. A mutual insurance contract contingent on a statistical state pays an individual a certain amount in a given individual state if and only if the economy as a whole is in a given statistical state.

Let $z_{jh\sigma}$ denote the quantity of good j consumed by household h in social state σ : $z_{h\sigma}$ is an N-dimensional vector of *all* goods consumed by h in social state σ, $z_{h\sigma} = z_{jh\sigma}$, $j = 1, \ldots, N$ and z_h is an NS^H-dimensional vector of *all goods consumed in all social states* by h, $z_h = z_{h\sigma}, \sigma \in \Omega$.[3]

Let $s(h, \sigma)$ be the state of individual h in the social state σ, and $r_s(\sigma)$ be the proportion of all households for whom $s(h, \sigma) = s$. Let $r(\sigma) = r_1(\sigma), \ldots, r_S(\sigma)$ be the distribution of households among individual states within the social state σ, i.e., the proportion of all individuals in state s for each s. $r(\sigma)$ is a statistical state. Let R be the set statistical states, i.e., of vectors $r(\sigma)$ when σ runs over Ω. R is contained in S^I, the product of I S-dimensional simplices, and has $\binom{H+S-1}{S-1}$ elements.

Π^h is household h's probability distribution over the set of social states Ω, and Π^h_σ denotes the probability of state σ. Although we take social states as the primitive concept, we in fact work largely with statistical states. We, therefore, relate preferences, beliefs and endowments to statistical states. This is done in the next section: clearly any distribution over social states implies a distribution over statistical states.

The following *anonymity assumption* is required:

$$r(\sigma) = r(\sigma') \rightarrow \Pi^h_\sigma = \Pi^h_{\sigma'}.$$

This means that two overall distributions σ and σ' which have the same statistical characteristics are equally likely. Then Π^h_σ defines a probability distribution Π^h_r on the space of statistical states R. Π^h_r can be interpreted, as remarked above, as h's distribution over possible distributions of impacts in the population as a whole. The probability that a statistical state r obtains and that simultaneously, for a given household h a particular state s also obtains, Π^h_{sr}, is[4]

$$\Pi^h_{sr} = \Pi^h_r r_s \quad \text{with} \quad \sum_s \Pi^h_{sr} = \Pi^h_r. \tag{1}$$

The probability Π^h_s that, for a given h, a particular individual state s obtains is, therefore, given by

$$\Pi^h_s = \sum_{r \in R} \Pi^h_r r_s,$$

where r_s is the proportion of people in individual state s in statistical state r. Note that we denote by $\Pi^h_{s|r}$ the conditional probability of household h being

in individual state s, conditional on the economy being in statistical state r. Clearly $\sum_s \Pi^h_{s|r} = 1$. Anonymity implies that

$$\Pi^h_{s|r} = r_s,$$

i.e., that the probability of anyone being in individual state s contingent on the economy being in statistical state r is the relative frequency of state s contingent on statistical state r.

3. The Behavior of Households

Let e^h_s be the endowment of household h when the individual state is s. We assume that household h always has the same endowment in the individual state s, whatever the social state. We also assume that all households have the same endowment if they are in the same individual state: endowments differ, therefore, only because of differences in individual states. This describes the risks faced by individuals.

Individuals have von Neumann–Morgenstern utilities:

$$W^h(z_h) = \sum_\sigma \Pi^h_\sigma U^h(z_{h\sigma}).$$

This definition indicates that household h has preferences on consumption which may be represented by a "state separated" utility function W^h defined from elementary state-dependent utility functions.

We assume like Malinvaud [15] that *preferences are separable over statistical states*. This means that the utility of household h depends on σ only through the statistical state $r(\sigma)$. If we assume further that in state σ household h takes into account only its individual consumption, and what overall frequency distribution $r(\sigma)$ appears, and nothing else, then its consumption plan can be expressed as $z^h_\sigma = z_{hsr}$: its consumption depends only on its individual state s and the statistical state r. Summation with respect to social states σ in the expected utility function can now be made first within each statistical state. Hence we can express individuals' utility functions as:

$$W^h(z_{h\sigma}) = \sum_{r,s} \Pi^h_{sr} U^h(z_{hsr}), \tag{2}$$

which expresses the utility of a household in terms of its consumption at individual state s within a statistical state r, summed over statistical states. This expression is important in the following results, because it allows us to represent the utility of consumption across social states σ as a function of statistical states r and individual states s only. The functions U^h_s are assumed to be C^2, strictly increasing, strictly quasiconcave, and the closure of the indifference surfaces $\{U^h_s\}^{-1}(x) \subset \mathrm{int}(R^{N+})$ for all $x \in R^+$. The probabilities Π^h_σ are in principle different over households.

4. Efficient Allocations

Let p^* be a competitive equilibrium price vector of the Arrow–Debreu economy E with markets contingent on all social states[5] and let z^* be the associated allocation. We will as usual say that z^* is *Pareto efficient* if it is impossible to find an alternative feasible allocation which is preferred by at least one agent and to which no agent prefers z^*. Let p^*_σ and z^*_σ be the components of p^* and z^*, respectively, which refer to goods contingent on state σ.

We now define an Arrow–Debreu economy E, where markets exists contingent on an exhaustive description of all states in the economy, i.e. for all social states $\sigma \in \Omega$. We, therefore, have NS^H contingent markets. An *Arrow–Debreu equilibrium* is a price vector $p^* = (p_\sigma), p_\sigma \in R^{N+}, \sigma \in \Omega$, and an allocation z^* consisting of vectors $z^*_h = (z^*_{h\sigma}), z^*_{h\sigma} \in R^{N+}, \sigma \in \Omega, h = 1, \ldots, H$ such that for all h, z^*_h maximizes

$$W^h(z^*_h) = \sum_\sigma \Pi^h_\sigma U^h(z^*_{h\sigma}) \tag{3}$$

subject to a budget constraint

$$p(z^*_h - e_h) = 0 \tag{4}$$

and all markets clear:

$$\sum_h (z^*_h - e_h) = 0. \tag{5}$$

Proposition 1 considers the case when households agree on the probability distribution over social states, this common probability being denoted by Π. It follows that they agree on the distribution over statistical states. It shows that in this case, the competitive equilibrium prices p^* and allocations z^* are the same across all social states σ leading to the same statistical state r.[6]

PROPOSITION 1. *When agents have common probabilities, i.e.,* $\Pi^h = \Pi^j$ $\forall h, j$, *then equilibrium prices depend only on statistical states. Consider an Arrow–Debreu equilibrium of the economy* E, $p^* = (p^*_\sigma)$, $z^* = (z^*_\sigma)$, $\sigma \in \Omega$. *For every state* σ *leading to a given statistical state* r, *i.e., such that* $r(\sigma) = r$, *equilibrium prices and consumption allocations are the same, i.e., there exists a price vector* p^*_r *and an allocation* z^*_r *such that* $\forall \sigma : r(\sigma) = r$, $p^*_\sigma = p^*_r$ *and* $z^*_\sigma = z^*_r$, *where* $p^*_r \in R^{N+}$ *and* $z^*_r \in R^{NI}$ *depend solely on* r.

Proof. In the Appendix.

DEFINITION. An economy E is *regular* if at all equilibrium prices in E the Jacobian matrix of first partial derivatives of its excess demand function has full rank [11]. Regularity is a generic property [10, 11].

We now consider the general case, which allows for $\Pi^h \neq \Pi^j$ if $h \neq j$. Proposition 1 no longer holds: the reason is that households may not achieve

full insurance at an equilibrium. However, Proposition 2 states that if the economy is *regular, if all households have the same preferences* and if there are two individual states, there is always one equilibrium at which prices are the same at all social states leading to the same statistical state. This confirms the intuition that the characteristics of an equilibrium should not be changed by a permutation of individuals: if I am changed to your state, and you to mine, everyone else remaining constant, then provided you and I have the same preferences, the equilibrium will not change.

PROPOSITION 2. *An Arrow–Debreu equilibrium allocation of the economy $E(p^*, z^*)$ is not fully insured if $\Pi^h \neq \Pi^k$ for some households h, k with $U^h \neq U^k$ in (2). In particular, household h has a different equilibrium allocation across social states σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$. When E is a regular economy, all agents have the same utilities,[7] and there are two individual states, then one of the equilibrium prices p^* must satisfy $p^*_{\sigma_1} = p^*_{\sigma_2}$ for all σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$.*

Proof. In the Appendix.

5. Equilibrium in Incomplete Markets for Unknown Risks

Consider first the case where *there are no assets to hedge against risk*, so that the economy has incomplete asset markets. Individuals cannot transfer income to the unfavorable states. Examples are cases when individuals are not able to purchase hurricane insurance, as in some parts of the south eastern United States and in the Caribbean. Market allocations are typically inefficient in this case, since individuals cannot transfer income from one state to another to equalize welfare across states. Which households will be in each individual state is unknown. Each individual has a certain probability distribution over all possible social states σ, Π^h. In each social state σ each individual is constrained in the value of her/his expenditures by her/his endowment (which depends on the individual state $s(h, \sigma)$ in that social state). In this context, a *general equilibrium of the economy with incomplete markets* E_I consists of a price vector p^* with NS^H components and H consumption plans z^*_h with NS^H components each, such that z^*_h maximizes $W^h(z_h)$:

$$W^h(z_h) = \sum_{\sigma} \Pi^h_\sigma U^h(z_{h\sigma}) \tag{6}$$

subject to

$$p_\sigma(z_{h\sigma} - e_{h\sigma}) = 0 \quad \text{for each} \quad \sigma \in \Omega \tag{7}$$

and

$$\sum_{h=1}^{H}(z_h - e_h) = 0. \tag{8}$$

The above economy E_I is an extreme version of an economy with incomplete asset markets (see, e.g., [13]) because there are no markets to hedge against risks: there are S^H budget constraints in (7).

6. Efficient Allocations, Mutual Insurance and Securities

In this section we study the possibility of supporting Arrow–Debreu equilibria by combinations of statistical securities and insurance contracts, rather than by using state contingent contracts. As already observed, this leads to a very significant economy in the number of markets needed. In an economy with no asset markets at all, such as E_I, the difficulty in supporting an Arrow–Debreu equilibrium arises because income cannot be transferred between states. On the basis of Propositions 1 and 2, we show that households can use securities defined on statistical states to transfer into each such state an amount of income equal to the expected difference between the value of Arrow–Debreu equilibrium consumption and the value of endowments in that state. The expectation here is over individual states conditional on being in a given statistical state. The difference between the actual consumption-income gap given a particular individual state and its expected value is then covered by insurance contracts. Recall that A is the binomial number $A = \binom{H+S-1}{S-1}$.

THEOREM 1. *Assume that all households in E have the same probability* Π *over the distribution of risks in the population. Then any Arrow–Debreu equilibrium allocation* (p^*, z^*) *of E (and, therefore, any Pareto Optimum) can be achieved within the general equilibrium economy with incomplete markets* E_I *by introducing a total of* $I.A$ *mutual insurance contracts to hedge against individual risk, and* A *statistical securities to hedge against social risk. In a regular economy with two individual states and identical preferences, even if agents have different probabilities, there is always an Arrow–Debreu equilibrium* (p^*, z^*) *in E which is achievable within the incomplete economy* E_I *with the introduction of* $I.A$ *mutual insurance contracts and A statistical securities.*

Proof. In the Appendix.

6.1. *Market Complexity*

We can now formalize a statement made before about the efficiency of the institutional structure proposed in Theorem 1 by comparison with the standard Arrow–Debreu structure of a complete set of state-contingent markets. We use here complexity theory, and in particular the concept of *NP-completeness*. The key consideration in this approach to studying problem complexity is how fast the number of operations required to solve a problem increases with the size of the problem.

DEFINITION. If the number of operations required to solve a problem must increase exponentially for any possible way of solving the problem, then the problem is called *"intractable"* or more formally, *NP-complete*. If this number increases polynomially, the problem is *tractable*. Further definitions are in [12].

The motivation for this distinction is of course that if the number of operations needed to solve the problem increases exponentially with some measure of the size of the problem, then there will be examples of the problem that no computer can or ever could solve. Hence there is no possibility of ever designing a general efficient algorithm for solving these problems. However, if the number of operations rises only polynomially then it is in principle possible to devise a general and efficient algorithm for the problem.

Theorem 2 investigates the complexity of the resource allocation problem in the Arrow–Debreu framework and compares this with the framework of Theorem 1. We focus on how the problem changes as the economy grows in the sense that the number of households increases, and consider a very simple aspect of the allocation problem, which is as follows. Suppose that the excess demand of the economy $Z(p)$ is known. A particular price vector p^* is proposed as a market clearing price. We wish to check whether or not it is a market clearing price. This involves computing each of the coordinates of $Z(p)$ and then comparing with zero. This involves a number of operations proportional to the number of components of $Z(p)$; we, therefore, take the rate at which the dimension of $Z(p)$ increases with the number of agents to be a measure of the complexity of the resource allocation problem. In summary: we ask how the difficulty of verifying market clearing increases as the number of households in the economy rises. We show that in the Arrow–Debreu framework this difficulty rises exponentially, whereas in the framework of Theorem 1 it rises only polynomially.

THEOREM 2. *Verifying market clearing is an intractable problem in an Arrow–Debreu economy, i.e., the number of operations required to check if a proposed price is market clearing increases exponentially with the number of households H. However, under the assumptions of Theorem 1, in the economy E_I supplemented by $I.A$ mutual insurance contracts and A statistical securities, verifying market clearing is a tractable problem, i.e., the number of operations needed to check for market clearing increases only polynomially with the number of households.*

Proof. The number of operations required to check that a price is market clearing is proportional to the number of market clearing conditions. In E we have NS^H markets. Hence the number of operations needed to check if a proposed price is market clearing must rise exponentially with the number of households H. Consider now the case of E_I supplemented by $I.A$ mutual insurance contracts and A securities. Under the assumptions of Theorem 1,

by Propositions 1 and 2, we need only check for market clearing in one social state associated with any statistical state, as if markets clear in one social state leading to a certain statistical state they will clear in all social states leading to the same statistical state. Hence we need to check a number of goods markets equal to $N.A$, plus markets for mutual insurance contracts and securities. Now

$$A = \binom{H+S+1}{S-1} = \Phi(H,S),$$

where $\Phi(H,S)$ is a polynomial in H of order $(S-1)$. Hence A itself is a polynomial in H whose highest order term depends on H^{S-1}, completing the proof. □

7. Catastrophe Futures and Bundles

We mentioned in the introduction that securities contingent on statistical states are already traded as "catastrophe futures" on the Chicago Board of Trade, where they were introduced in 1994. Recently, hurricane bonds and earthquake bonds have been introduced, additional examples of statistical securities. (The concept was discussed by Chichilnisky and Heal in 1993 [6].) Catastrophe futures are securities which pay an amount that depends on the value of an index of insurance claims paid during a year. One such index measures the value of hurricane damage claims: others measure claims stemming from different types of natural disasters. The value of hurricane damage claims depends on the overall incidence of hurricane damage in the population, but is not of course affected by whether any particular individual is harmed. It, therefore, depends, in our terminology, on the statistical state, on the distribution of damage in the population, but not on the social state. Catastrophe futures are thus financial instruments whose payoffs are conditional on statistical state of the economy: they are statistical securities. According to our theory, a summary version of which appeared in [6] in 1993, they are a crucial prerequisite to the efficient allocation of unknown risks. And as the incidence and extent of natural disaster claims in the U.S. has increased greatly in recent years, risks such as hurricane risks are in effect unknown risks: insurers are concerned that the incidence of storms may be related to trends in the composition of the atmosphere and incipient greenhouse warming. However, catastrophe futures are not on their own sufficient for this: they do not complete the market. Mutual insurance contracts, as described above, are also needed. These provide insurance conditional on the value of the catastrophe index. The two can be combined into "catastrophe bundles", see [3].

8. Conclusions

We have defined an economy with unknown individual risks, and established that a combination of statistical securities and mutual insurance contracts can be used to obtain an efficient allocation of risk-bearing. Furthermore, we have shown that this institutional structure is efficient in the sense that it requires exponentially fewer markets that the standard approach via state-contingent commodities. In fact, the state-contingent problem is "intractable" with individual risks (formally, NP-complete) in the language of computational complexity, whereas our approach gives a formulation that is polynomially complex. This greatly increases the economy's ability to achieve efficient allocations. Another interesting feature of this institutional structure is the interplay of insurance and securities markets involved. Its simplicity leads to successful hedging of unknown risks and predicts some convergence between the insurance and securities industries.

9. Appendix

PROPOSITION 1. *When agents have common probabilities, i.e.,* $\Pi^h = \Pi^j$ $\forall h, j$. *then equilibrium prices depend only on statistical states. Consider an Arrow–Debreu equilibrium of the economy* E, $p^* = (p^*_\sigma)$, $z^* = (z^*_\sigma)$, $\sigma \in \Omega$. For every state σ *leading to a given statistical state* r, *i.e., such that* $r(\sigma) = r$, *equilibrium prices and consumption allocations are the same, i.e., there exists a price vector* p^*_r *and an allocation* z^*_r *such that* $\forall \sigma : r(\sigma) = r$, $p^*_\sigma = p^*_r$ *and* $z^*_\sigma = z^*_r$, *where* $p^*_r \in R^{N+}$ *and* $z^*_r \in R^{NI}$ *depend solely on* r.

Proof. Consider σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2) = r$. Note that the total endowments of the economy are the same in σ_1 and σ_2, both equal to $s_r = Hr_s e_{hs}$ (recall that $e_{hs} = e_s$ as endowments depend only on individual states and not on household identities). Also, by the anonymity assumption, $\Pi_{\sigma_1} = \Pi_{\sigma_2} = \Pi_r$, where Π_r is the common probability of any social state in the statistical state r. Let $\Pi_{\sigma|r}$ be the probability of being in social state σ given statistical state r. By the anonymity assumption on probabilities this is just $1/\#\Omega_r$. We now show that for every household h, $z^*_{h\sigma_1} = z^*_{h\sigma_2}$, due to the Pareto efficiency of Arrow–Debreu equilibria. Let $\Omega_r = \{\sigma : r(\sigma) = \sigma\}$. Let $z^* = (z^*_{h\sigma})$, and assume in contradiction to the proposition that there are σ_1 and $\sigma_2 \in \Omega_r$ such that $z^*_{h\sigma_1} \neq z^*_{h\sigma_2}$. for some h. Define $Ez_{hr} = \sum_{\sigma\in\Omega_r} z^*_{h\sigma}\Pi_{\sigma|r} = (1/\#\Omega_r)\sum_{\sigma\in\Omega_r z^*_{h\sigma}}$. This is the expected value of $(z^*_{h\sigma})$ given that the economy is in the statistical state r. Now

$$\sum_h Ez_{hr} = \sum_h \frac{1}{\#\Omega_r} \sum_{\sigma\in\Omega_r} z^*_{h\sigma} = \sum_h z^*_{h\sigma},$$

so that $Ez_{h\sigma}$ is a feasible consumption vector for each h in the statistical state r. Next we show that by strict concavity, moving for each h and each σ from

$z^*_{h\sigma}$ (which depends on σ) to Ez_{hr} (which is the same for all $\sigma \in \Omega$), is a strict Pareto improvement. This is because

$$W^h(z^*_{h\sigma}) = \sum_\sigma \Pi_\sigma U^h(z^*_{h\sigma}) = \sum_r \Pi_r \sum_{\sigma\in\Omega} \Pi_{\sigma|r} U^h(z^*_{h\sigma}).$$

By strict concavity of preferences,

$$\sum_r \Pi_r \sum_{\sigma\in\Omega_r} \Pi_{\sigma|r} U^h(z^*_{h\sigma})$$

$$< \sum_r \Pi_r \sum_{\sigma\in\Omega_r} U^h\left(\sum_{\sigma\in\Omega} z^*_{h\sigma}\Pi_{\sigma|r}\right) = \sum_r \Pi_r \sum_{\sigma\in\Omega} U^h(Ez_{h\sigma}).$$

Since $Ez_{h\sigma}$ is Pareto superior to z^* with $z^*_{h\sigma_1} \neq z^*_{h\sigma_2}$, such a z^* cannot be an equilibrium allocation. Hence $z^*_{h\sigma_1} = z^*_{h\sigma_2} = z^*_{hr}$ for all $h = 1, \ldots, H$. Note that this implies that in an equilibrium, household h consumes the same allocation z^*_{hr} across all individual states s in a given statistical state, i.e. it achieves full insurance. Since p^* supports the equilibrium allocation z^*, and $z^*_{h\sigma_1} = z^*_{h\sigma_2}$ it follows that $p^*_{\sigma_1} = p^*_{\sigma_2}$ when $r(\sigma_1) = r(\sigma_2)$, because utilities are assumed to be C^2 and, in particular, to have a unique gradient at each point which, by optimality, must be collinear both with $p^*_{\sigma_1}$ and with $p^*_{\sigma_2}$, i.e. $p^*_{\sigma_1} = p^*_{\sigma_2} = p^*_r$. This implies that at an equilibrium, household h faces the same prices p^*_r at any σ with $r(\sigma) = r$. □

PROPOSITION 2. *An Arrow–Debreu equilibrium allocation of the economy $E(p^*, z^*)$ is not fully insured if $\Pi^h \neq \Pi^k$ for some households h, k with $U^h \neq U^k$. In particular, household h has a different equilibrium allocation across social states σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$. When E is a regular economy, all agents have the same utilities,*[8] *and there are two individual states, one of the equilibrium prices p^* must satisfy $p^*_{\sigma_1} = p^*_{\sigma_2}$ for all σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$.*

Proof. Suppose that household h is in fact fully insured so that $z^*_{h\sigma_1} = z^*_{h\sigma_2}$ for all σ^1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$. Household h's consumption levels are $y^i_{s_1r}$ and $y^i_{s_2r}$ where $s_1 = s(h, \sigma_1)$ and $s_2 = s(h, \sigma_2)$. By assumption we have $y^i_{s_1r} = y^i_{s_2r}$. Now from (2) household h's marginal rate of substitution between consumption in states σ_1 and σ_2 is $\Pi^h_{s_1|r}/\Pi^h_{s_2|r}$. Suppose also that household k, $k \neq h$, is fully insured. Then by the same argument k's marginal rate of substitution between consumption in states σ_1 and σ_2 is $\Pi^k_{s_1|r}/\Pi^k_{s_2|r}$. But if different households have different probability distributions this is a contradiction as both face the same price vector.

Assume now that E is regular, that all agents have the same preferences, and that $S = 2$. Consider two social states σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$, and such that σ_1 differs from σ_2 only on the individual states of the two households h_1 and h_2 which are permuted, i.e., $s(h_1, \sigma_1) = s(h_2, \sigma_2)$ and

$s(h_2, \sigma_1) = s(h_1, \sigma_2)$. Assume that there exists an equilibrium price for E, $p^* \in R^{NS^H}$, such that its components in states σ_1 and σ_2 are different, i.e. $p^*_{\sigma_1} \neq p^*_{\sigma_2}$. Define now a new price $p^*_c \in R^{NS^H}$, called a "conjugate" of p^*, which differs from p^* only in its coordinates in states σ_1 and σ_2, which are permuted as follows: $\forall\, \sigma \neq \sigma_1, \sigma_2, \bar{p}^*_\sigma = p^*_\sigma, \bar{p}^*_{\sigma_1} = p^*_{\sigma_2}$, and $\bar{p}^*_{\sigma_2} = p^*_{\sigma_1}$. We shall now show that $\bar{p}^*$ is also an equilibrium price for the economy E. At $\bar{p}^*$, household h_1 has the same endowments and faces the same prices in states σ_1 and σ_2 as it did at states σ_2 and σ_1 respectively at price p^*; at all other states $\sigma \in \Omega$, h_1 faces the same prices and has the same endowments facing p^* and facing $\bar{p}^*$. The same is true of household h_2. Furthermore, h_1 and h_2 have the same utilities and probabilities at σ_1 and σ_2 because $r(\sigma_1) = r(\sigma_2)$ and probabilities are anonymous. Therefore, the excess demand vectors of h_1 in states σ_1 and σ_2 at prices p^* equal the excess demand vectors of h_2 in σ_2 and σ_1 respectively, at prices $\bar{p}^*$, and at all other states $\sigma \in \Omega$ the excess demand vectors of h_1 are the same at prices p^* and $\bar{p}^*$. Reciprocally: the excess demand vectors of h_2 in σ_1 and σ_2 at prices p^* equal the excess demand vectors of h_1 in σ_2 and σ_1 respectively at prices $\bar{p}^*$, and in all other states σ, the excess demand vectors of h_2 are the same as they are with prices p^*. Formally:

$$z_{h_1\sigma_1}(\bar{p}^*) = z_{h_2\sigma_2}(p^*), z_{h_1\sigma_2}(\bar{p}^*) = z_{h_2\sigma_1}(p^*)$$
$$z_{h_2\sigma_1}(\bar{p}^*) = z_{h_1\sigma_2}(p^*), z_{h_2\sigma_2}(\bar{p}^*) = z_{h_1\sigma_1}(p^*)$$

and $\forall \sigma \in \Omega, \sigma \neq \sigma_1, \sigma_2$:

$$z_{h_1\sigma}(p^*) = z_{h_1\sigma}(\bar{p}^*), z_{h_2\sigma}(p^*) = z_{h_2\sigma}(\bar{p}^*).$$

The excess demand vectors of all other households $h \neq h_1, h_2$ are the same for p^* and $\bar{p}^*$. Therefore, at $\bar{p}^*$ the aggregate excess demand vector of the economy is zero, so that $\bar{p}^*$ is an equilibrium. The same argument shows that permuting the two components $p^*_{\sigma_1}, p^*_{\sigma_2}$ of a price p^* at any two social states σ_1, σ_2 leading to the same statistical state $r(\sigma_1)$ leads from an equilibrium price p^* to another equilibrium price $\bar{p}^*$. This is because if two social states σ_1 and σ_2 lead to the same statistical state and there are two individual states s_1 and s_2 then there is a number $k > 0$ such that k households who are in s_1 in σ_1 are in s_2 in σ_2 and another k households who were in s_1 in σ_2 are in s_2 in σ_1, while remaining in the same individual states otherwise. These two sets of k households can be paired. For every pair of households, the above argument applies. Hence it applies to the sum of the demands, so that the new price $\bar{p}^*$ is an equilibrium.

Now consider any regular economy E with a finite number of equilibrium prices denoted $p^*_1, \ldots, p^*_k$. We shall show that there exists a $j \leq k$ s.t. p^*_j assigns the same price vector to all social states σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$. Start with p^*_1 : if p^*_1 does not have this property, consider the first two social states σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$ and $p^*_{1\sigma_1} \neq p^*_{1\sigma_2}$. Define $\bar{p}^*_1$ as the conjugate of p^*_1 constructed by permuting the prices of the social states σ_1 and σ_2. If

$\forall j > 1, p_j^* = \bar{p}_1^*$, then there are two price equilibria, i.e. $k = 2$; however, since the number of price equilibria must be odd,[9] there must exist $p_{j_1}^*$ with $j_1 > 1$, and $p_{j_1}^* \neq \bar{p}_1^*$. Consider now the conjugate of $p_{j_1}^*$ with respect to the first two social states σ_1, σ_2 which correspond to the same statistical state and have different components in $p_{j_1}^*$, and denote this conjugate $\bar{p}_{j_1}^*$. Repeat the procedure until all equilibria are exhausted. In each step of this procedure, two different price equilibria are found. Since the number of equilibria must be odd, it follows that there must exist a $j \leq k$ for which all conjugates of p_j^* equal p_j^*: this is the required equilibrium which assigns the same equilibrium prices $p_{\sigma_1}^* = p_{\sigma_2}^*$ to all σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$, completing the proof. □

THEOREM 1. *Assume that all households in E have the same probability Π over the distribution of risks in the population. Then any Arrow–Debreu equilibrium allocation* (p^*, z^*) *of E (and, therefore, any Pareto Optimum) can be achieved within the general equilibrium economy with incomplete markets* E_I *by introducing a total of I.A mutual insurance contracts to hedge against individual risk, and A statistical securities to hedge against social risk. In a regular economy with two individual states and identical preferences, even if agents have different probabilities, there is always an Arrow–Debreu equilibrium* (p^*, z^*) *in E which is achievable within the incomplete economy* E_I *with the introduction of I.A mutual insurance contracts and A statistical securities.*

Proof. Consider first the case where all households have the same probabilities, i.e., $\Pi^h = \Pi^j = \Pi$. By Proposition 1, an Arrow–Debreu equilibrium of E has the same prices $p_\sigma^* = p_r^*$ and the same consumption vectors $z_{h\sigma}^* = z_{hr}^*$ for each h, at each social state σ with $r(\sigma) = r$. Define $\Omega(r)$ as the set of social states mapping to a given statistical state r, i.e. $\Omega(r) = \{\sigma \in \Omega : r(\sigma) = r\}$. The budget constraint (4) is

$$p^*(z_h^* - e_h) = \sum_\sigma p_\sigma^* \left(z_{h,\sigma}^* - e_{h\sigma}\right) = \sum_r p_r^* \sum_{\sigma \in \Omega(r)} (z_{h\sigma}^* - e_{h\sigma}) = 0.$$

Individual endowments depend on individual states and not on social states, so that $e_{h\sigma} = e_{hs(\sigma)} = e_{hs}$; furthermore, by Proposition 1 equilibrium prices depend on r and not on σ, so that for each r the equilibrium consumption vector $z_{h\sigma}$ can be written as z_{hs}. The individual budget constraint is, therefore, $\sum_r p_r^* \sum_{s(r)} (z_{hs} - e_{hs})$, where summation over $s(r)$ indicates summation over all individual states s that occur in any social state leading to r, i.e. that are in the set $\Omega(r)$. Let $\#\Omega(r)$ be the number of social states in $\Omega(r)$. As $\Pi_{s|r} = r_s$ is the proportion of households in state s within the statistical state r, we can finally rewrite the budget constraint (4) of the household h as:

$$\#\Omega(r) \sum_r p_r^* \sum_s \#\Omega(r)\, \Pi_{s|r} (z_{hs} - e_{hs}) = 0. \tag{9}$$

Using (2), the household's maximization problem can, therefore, be expressed as:

$$\max \sum_{s,r} \Pi_{sr} U^h (z_{hsr}) \text{ subject to (9)}$$

and the equilibrium allocation z_h^* by definition solves this problem. Similarly, we may rewrite the market clearing condition (5) as follows:

$$\sum_h (z_h^* - e_h) = \sum_h (z_{h\sigma}^* - e_{hs(\sigma)}) = 0, \quad \forall \sigma \in \Omega.$$

Rewriting the market clearing condition (5) in terms of statistical states r, and within each r, individual states s, we obtain:

$$\sum_s r_s H \left(z_{hr}^* - e_{sr}^h \right) = 0, \quad \forall r \in R \tag{10}$$

or equivalently:

$$\sum_s \Pi_{s|r} H \left(z_{hr}^* - e_{sr}^h \right) = 0, \quad \forall r \in R.$$

Using these relations, we now show that any Arrow–Debreu equilibrium allocation $z^* = (z_{hr}^*)$ is within the budget constraints (7) of the economy E_I for each $\sigma \in \Omega$, *provided that* for each $\sigma \in \Omega$ we add the income derived from a statistical security $A_r, r = r(\sigma)$, and, given $r(\sigma)$, the income derived from mutual insurance contracts $m_{sr}^h = m_{s(\sigma)r(\sigma)}^h, s = 1, \ldots, S$. We introduce A statistical securities and $I.A$ mutual insurance contracts in the general equilibrium economy with incomplete markets E_I. The quantity of the security A_r purchased by household h in statistical state r, when equilibrium prices are p^*, is:

$$a_r^{h*} = \sum_s \Pi_{s|r} p_r^* (z_{hr}^* - e_{hs}). \tag{11}$$

The quantity a_r^{h*} has a very intuitive interpretation. It is the expected amount by which the value of equilibrium consumption exceeds the value of endowments, conditional on being in statistical state r. So *on average*, the statistical securities purchased deliver enough to balance a household's budget in each statistical state. Differences between the average and each individual state are taken care of by the mutual insurance contracts. Note that (10) implies that the total amount of each security supplied is zero, i.e., $\sum_h a_r^{h*} = 0$ for all r, so that this corresponds to the initial endowments of the incomplete economy E_I. Furthermore, $\sum_r a_r^{h*} = 0$ by (9), so that each household h is within her/his budget in E_I.

We now introduce a mutual insurance contract as follows. The transfer made by individual h in statistical state r and individual state s, when prices are p_r^*, is:

$$m_{sr}^{h*} = p_r^* (z_{hr}^* - e_{hr}) - a_r^{h*}. \tag{12}$$

Note that, as remarked above, m_{sr}^{h*} is just the difference between the actual income-expenditure gap, given that individual state s is realized, and the expected income-expenditure gap a_r^{h*} in statistical state r, which is covered by statistical securities. In each statistical state r, the sum over all h and s of all transfers m_{sr}^{h*} equals zero, i.e. the insurance premia match exactly the payments: for any given r,

$$\sum_{h,s} H\Pi_{s|r} m_{sr}^{h*} = \sum_{h,s} H\Pi_{s|r} p_r^* (z_{hr}^* - e_{hs}) - \sum_h H a_r^{h*} \sum_s \Pi_{s|r} = 0 \tag{13}$$

because $\sum_s \Pi_{s|r} = 1$. Therefore, the $\{m_{sr}^{h*}\}$ meet the definition of mutual insurance contracts. Finally, note that with N spot markets, A statistical securities $\{a_r\}$ and I mutual insurance contracts $\{m_{sr}^h\}$

$$p_r^* \left(z_{hr}^* - e_s^h\right) = m_{sr}^{i*} + a_r^{i*}, \quad \forall \sigma \in \Omega \text{ with } r(\sigma) = r, s = s(\sigma) \tag{14}$$

so that (7) is satisfied for each $\sigma \in \Omega$. This establishes that when all households have the same probabilities over social states, all Arrow–Debreu equilibrium allocation z^* of E can be achieved within the incomplete markets economy E_I when A securities and $I.A$ mutual insurance contracts are introduced into E_I, and completes the proof of the first part of the proposition dealing with common probabilities.

Consider now the case where the economy E is regular, different households in E have different probabilities over social states but have the same preferences, and $S = 2$. By Proposition 2, we know that within the set of equilibrium prices there is one p^* in which at all social states $\sigma \in \Omega(r)$ for a given r, the equilibrium prices are the same, i.e. $p_\sigma^* = p_r^*$. In particular, if E has a unique equilibrium (p^*, z^*), it must have this property. It follows from the above arguments that the equilibrium (p^*, z^*) must maximize (2) subject to (9). Now define the quantity of the security A_r purchased by a household in the statistical state r by

$$a_r^{h*} = \sum_s \Pi_{s|r}^h p_r^* \left(z_{hsr}^* - e_s^h\right) \tag{15}$$

and the mutual insurance transfer made by a household in statistical state r and individual state s, by

$$m_{sr}^{h*} = p_r^* \left(z_{hsr}^* - e_s^h\right) - a_r^{h*}. \tag{16}$$

As before, $\sum_r a_r^{h*} = 0$ and for any given r, $\sum_{h,s} \Pi_{s|r}^h H m_{sr}^{h*} = \sum_{h,s} r_s H m_{sr}^{h*} = 0$, so that the securities purchased correspond to the initial endowments of the economy E_I and at any statistical state the sum of the premia and the sum of the payments of the mutual insurance contracts match, completing the proof. □

Notes

1. A dealy viral disease.
2. Many were associated with hurricane Andrew which at $18 billion in losses was the most expensive catastrophe ever recorded. Some of the problems which beset Lloyds of London arose from underestimating environmental risks.
3. All consumption vectors are assumed to be non-negative.
4. See [16, p. 387, para. 1].
5. Defined formally below.
6. Related propositions were established by Malinvaud in an economy where all agents are identical, and risks are known.
7. The condition that all agents have the same preferences is not needed for this result. However, it simplifies that notation and the argument considerably. The general case is treated in the working papers from which this article derives.
8. The condition that all agents have the same preferences is not needed for this result, but simplifies the notation and the proof considerably. In the working papers from which this article derives, the general case was covered.
9. This follows from Dierker [11, p. 807] noting that his condition D is implied by our assumption that preferences are strictly increasing (see Dierker's remark following the statement of property D on p. 799).

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CLAIMS

1. A method for facilitating insuring/hedging against a risk condition, comprising establishing/using an index which is a measure for the condition.

2. The method of claim 1, wherein using is under a contract.

3. The method of claim 2, wherein the contract is a license contract.

4. The method of claim 1, wherein using is in a contract.

5. The method of claim 4, wherein the contract comprises an option contract.

6. The method of claim 4, wherein the contract comprises a futures contract.

7. The method of claim 1, wherein the condition is measured scientifically.

8. The method of claim 1, wherein the condition comprises a political condition.

9. The method of claim 1, wherein the condition comprises an environmental condition.

10. The method of claim 9, wherein the environmental condition comprises an atmospheric condition.

11. The method of claim 10, wherein the atmospheric condition comprises a temperature condition.

12. The method of claim 11, wherein the temperature condition comprises heating/cooling degree days (HDD/CDD) in a pre-specified geographic region over a

pre-specified time period.

13. The method of claim 9, wherein the environmental condition comprises an oceanographic condition.

14. The method of claim 13, wherein the oceanographic condition comprises an El Niño Southern Oscillator condition.

15. A method in insuring/hedging against a risk condition, comprising buying/selling a contract which is contingent on an index which is a measure for the risk condition.

16. The method of claim 15, wherein the contract comprises an option contract.

17. The method of claim 15, wherein the contract comprises a futures contract.

18. The method of claim 15, wherein the condition is measured scientifically.

19. The method of claim 15, wherein the condition is a political condition.

20. The method of claim 15, wherein the condition comprises an environmental condition.

21. The method of claim 20, wherein the environmental condition comprises an atmospheric condition.

22. The method of claim 21, wherein the atmospheric condition comprises a temperature condition.

23. The method of claim 22, wherein the temperature condition comprises

heating/cooling degree days (HDD/CDD) in a pre-specified geographic region over a pre-specified time period.

24. The method of claim 20, wherein the environmental condition comprises an oceanographic condition.

25. The method of claim 24, wherein the oceanographic condition comprises an El Niño Southern Oscillator condition.

26. A method in insuring/hedging against a risk, comprising, in combination:

buying/selling at least one insurance contract for a risk condition; and

buying/selling at least one security which is contingent on an index which is a measure for the risk condition;

wherein the combination yields a payment in an amount which depends on one or more triggers.

27. The method of claim 26, wherein the security is an option contract.

28. The method of claim 26, wherein the security is a futures contract.

29. The method of claim 26, wherein the security is priced as a function of probability of different catastrophic regimes and on incidence of loss in the regimes.

30. The method of claim 26, wherein one of the triggers is based on a correlated risk and another on an uncorrelated event.

31. The method of claim 30, wherein the uncorrelated event comprises a pre-specified scientific pattern.

32. The method of claim 30, wherein the uncorrelated event comprises a pre-specified political pattern.

33. The method of claim 30, wherein the uncorrelated event comprises a pre-specified environmental pattern.

34. The method of claim 33, wherein the environmental pattern comprises an atmospheric pattern.

35. The method of claim 34, wherein the atmospheric pattern comprises a temperature pattern in a pre-specified geographic region and over a pre-specified time period.

36. The method of claim 34, wherein the atmospheric pattern comprises a precipitation pattern in a pre-specified geographic region and over a pre-specified time period.

37. The method of claim 33, wherein the environmental pattern comprises an oceanographic pattern.

38. The method of claim 37, wherein the oceanographic pattern comprises an El Niño Southern Oscillator pattern.

HURRICANE PROBABILITIES AND THE ENSO SYSTEM

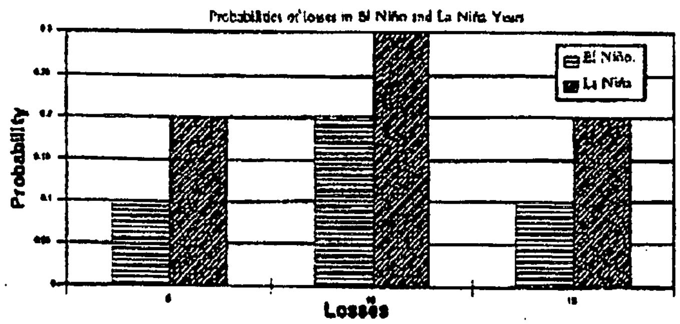

Fig. 1

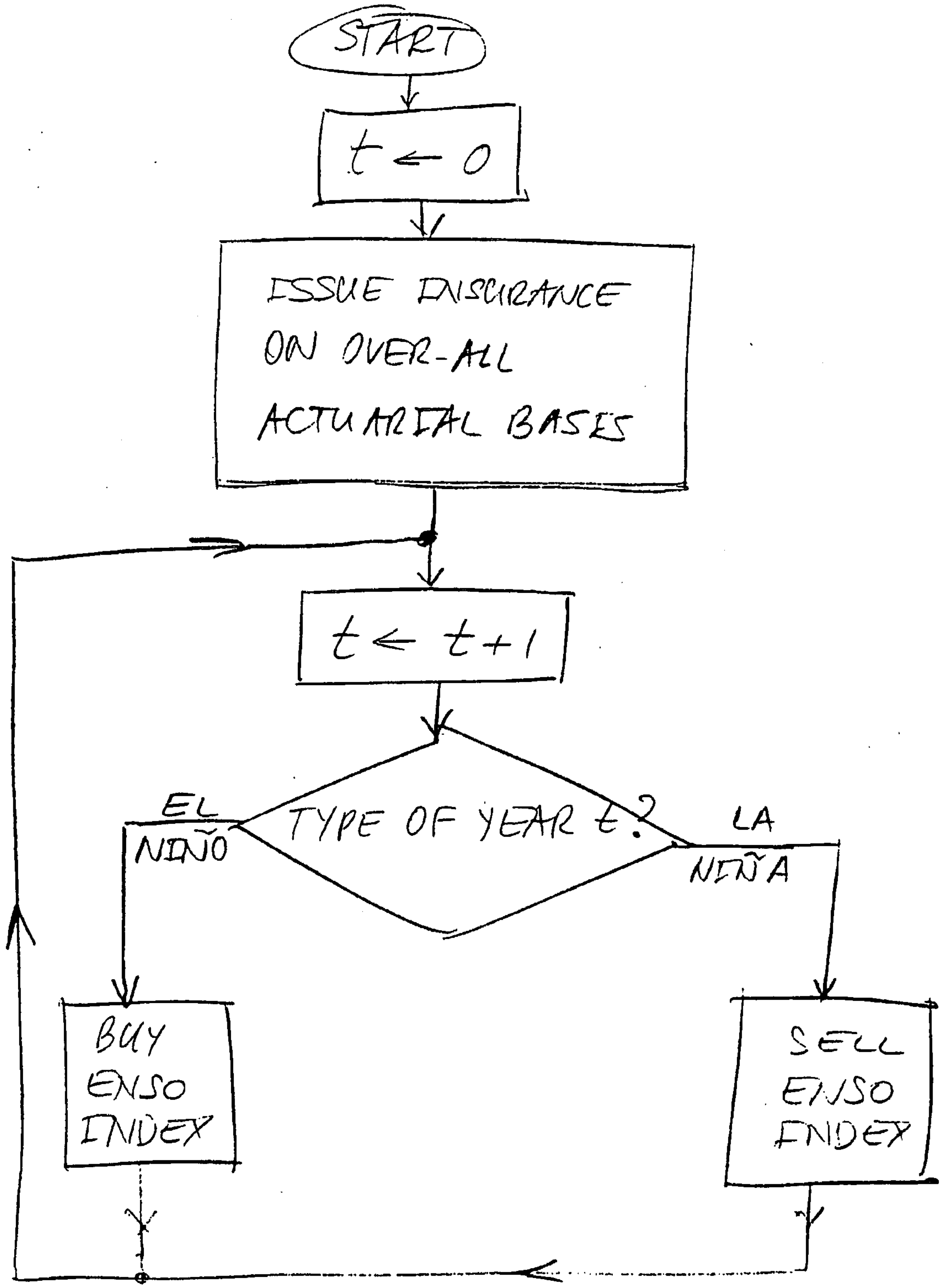
START
t ← 0
ISSUE INSURANCE ON OVER-ALL ACTUARIAL BASES
t ← t+1
TYPE OF YEAR t?
EL NIÑO
LA NIÑA
BUY ENSO INDEX
SELL ENSO INDEX

FIG. 2

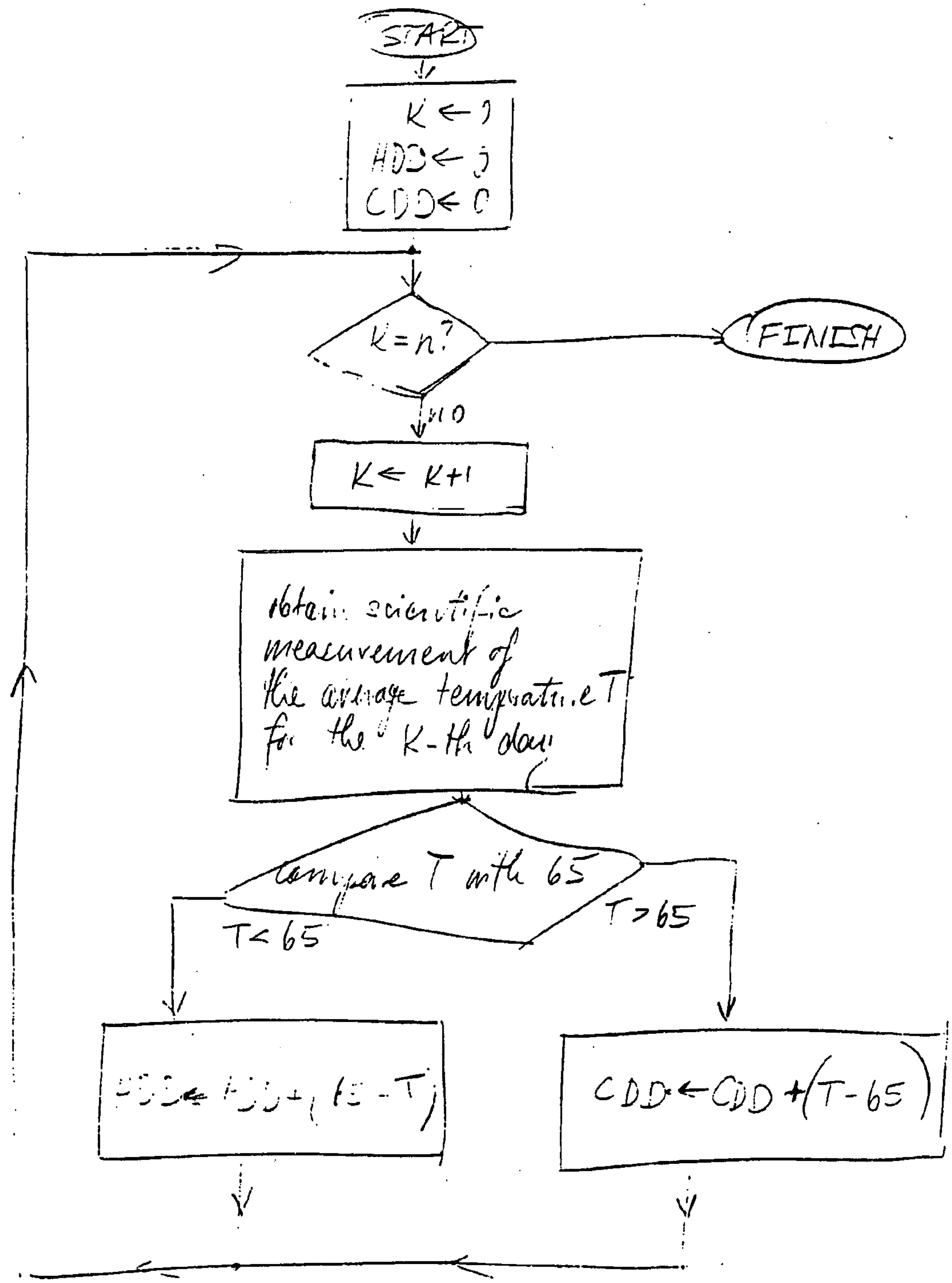
START
K ← 0
HDD ← 0
CDD ← 0
K = n?
FINISH
NO
K ← K+1
obtain scientific measurement of the average temperature T for the K-th day
compare T with 65
T < 65
T > 65
HDD ← HDD + (65 - T)
CDD ← CDD + (T - 65)

Fig. 3

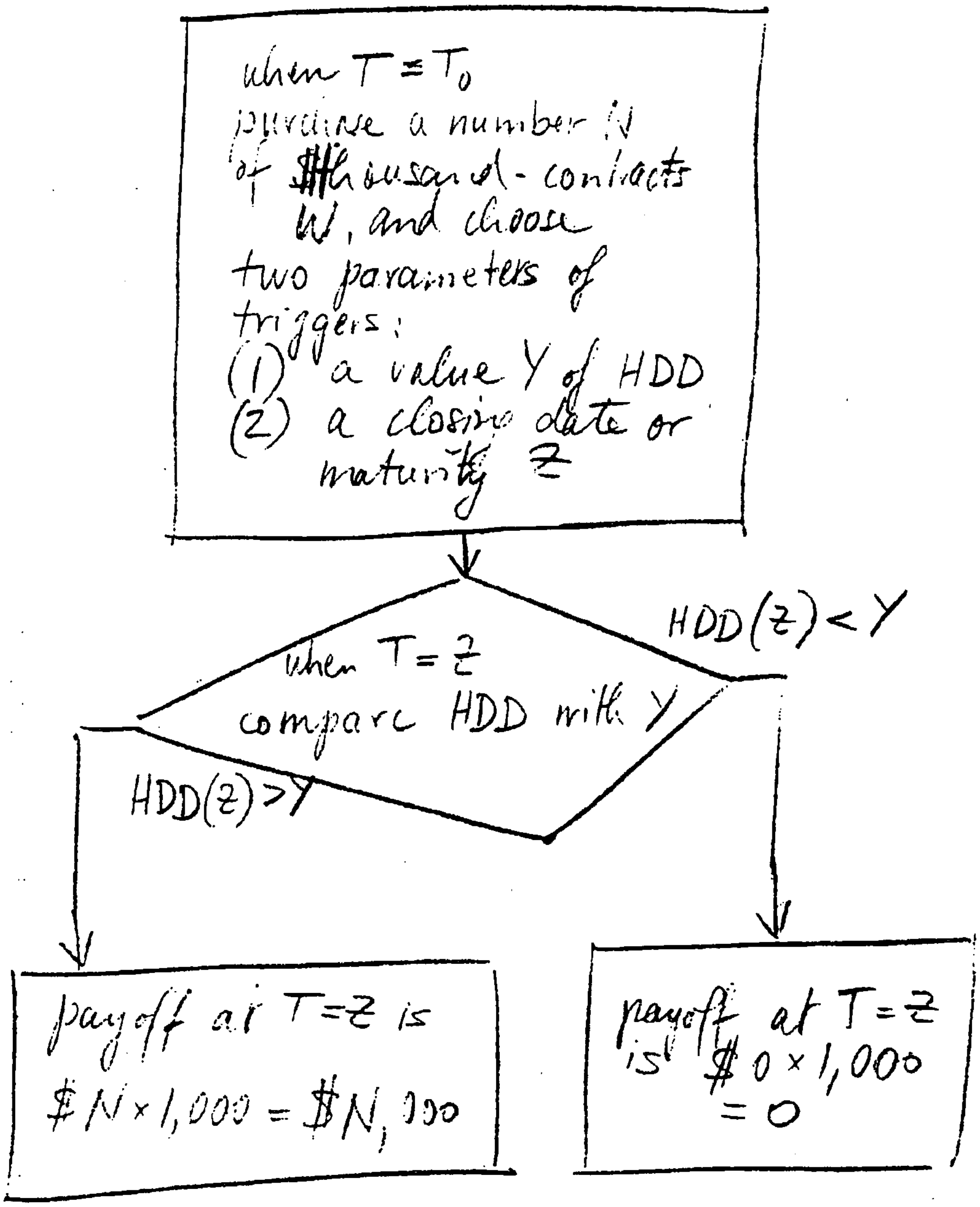
when T = T0
purchase a number N
of thousand-contracts
W, and choose
two parameters of
triggers:
(1) a value Y of HDD
(2) a closing date or
maturity Z
when T = Z
compare HDD with Y
HDD(Z) < Y
HDD(Z) > Y
payoff at T=Z is
$N × 1,000 = $N,000
payoff at T=Z
is $0 × 1,000
= 0

Fig. 4A

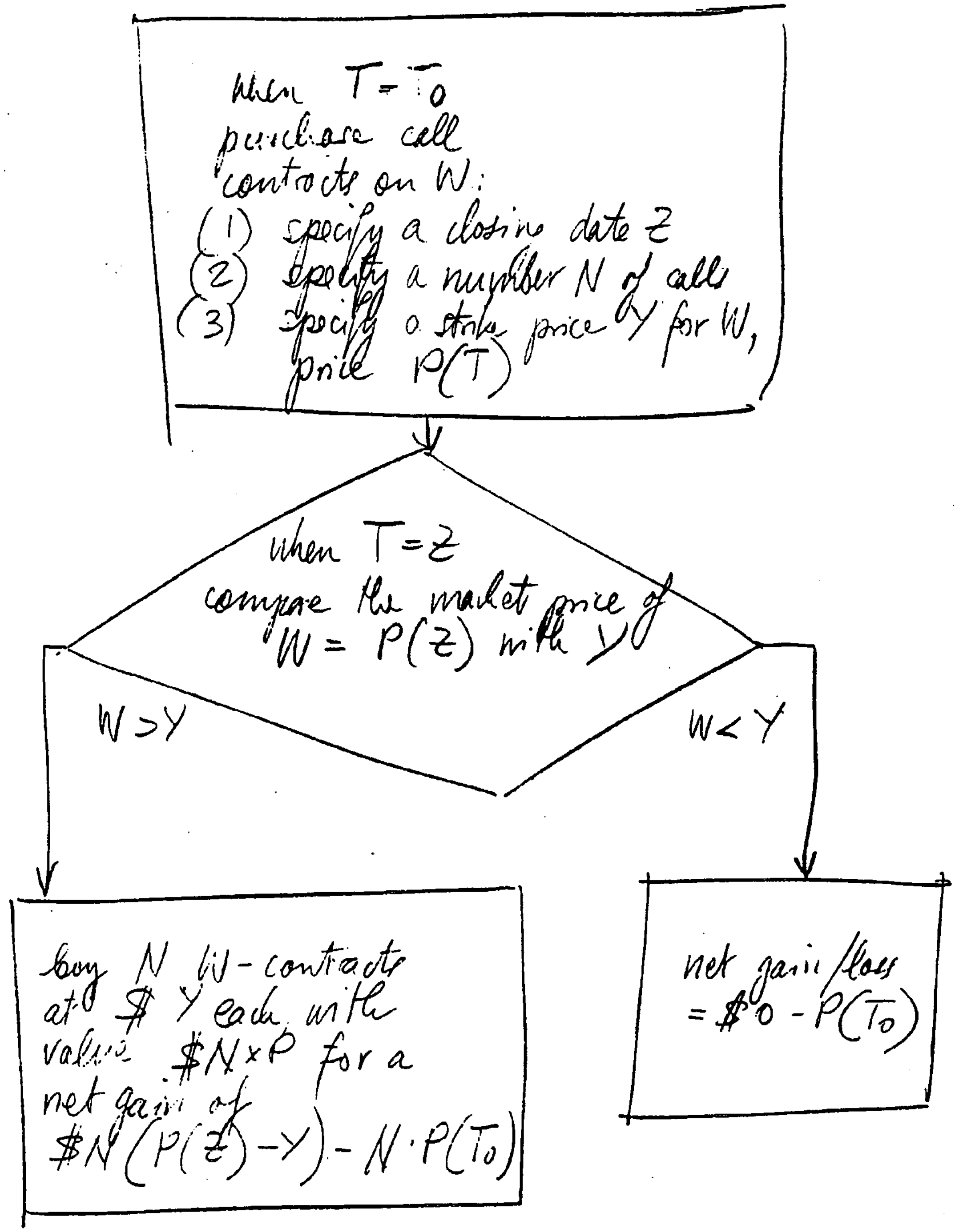
when T = T0
purchase call
contracts on W:
(1) specify a closing date Z
(2) specify a number N of calls
(3) specify a strike price Y for W,
price P(T)
when T = Z
compare the market price of
W = P(Z) with Y
W > Y
W < Y
buy N W-contracts
at $ Y each with
value $N×P for a
net gain of
$N(P(Z) - Y) - N·P(T0)
net gain/loss
= $0 - P(T0)

Fig. 4B

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US99/17709

A. CLASSIFICATION OF SUBJECT MATTER

IPC(6) : G06F 15/20, 15/21, 15/40, 17/60, 155/00, 157/00
US CL : 705/4
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. : 705/4

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X,P --- Y,P	US 5,884,274 A (WALKER et al) 16 March 1999, col 7, lines 8-61.	1, 3, 8, 30 ----------- 2, 4-7, 9-29, 31-36
Y,P	US 5,884,286 A (DAUGHTERY, III) 16 March 1999, col 12, lines 51-54.	5, 16, 27
Y,P	US 5,897,619 A (HARGROVE, JR. et al) 27 April 1999, col 11, line 40 - col 14, line 52.	7, 9-12, 18, 20-23, 31, 33-35
X,P	US 5,852,808 A (CHERNY) 22 December 1998, col 7, lines 37-42, col 9, lines 42-46, col 10, lines 8-24, col 11, lines 55-57.	1-2, 4-7, 15-18, 26-31

[X] Further documents are listed in the continuation of Box C. [ ] See patent family annex.

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"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
01 OCTOBER 1999	19 OCT 1999

Name and mailing address of the ISA/US	Authorized officer
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INTERNATIONAL SEARCH REPORT

International application No.
PCT/US99/17709

C (Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 5,752,237 A (CHERNY) 12 May 1998, col 7, lines 49-54, col 9, lines 50-54, col 10, lines 15-32, col 11, lines 62-67, col 13, lines 52-65.	1-2, 4-7, 15-18, 26-31
Y	US 5,202,827 A (SOBER) 13 April 1993, col 1, lines 32-46, col 5, lines 34-58.	2, 4-6, 8, 15-17, 19, 26-28, 32
X -- Y	US 4,766,539 A (FOX) 23 August 1988, col 1, lines 57-60, col 3, line 9 - col 4, line 35, col 7, lines 11-30.	37-38 ----------- 7, 9-14, 18, 20-25, 29, 31, 33-36